# Transverse Radial Expansion and Directed Flow

Sergei A. Voloshin*

*Physikalisches Institut der Universität Heidelberg, Heidelberg, Germany*

(February 9, 2008)

The effects of an interplay of radial expansion of the thermalized system created in a heavy ion collision and directed flow are discussed. It is shown that the study of azimuthal anisotropy of particle distribution as a function of rapidity *and* transverse momentum could reveal important information on both radial and directed flow.

Recently, the study of collective flow in nuclear collisions at high energies has attracted an increased attention of both theoreticians and experimentalists. There are several reasons for that: i) the observation of anisotropic flow at the AGS [1,2] and, probably, at the SPS [3] energies, ii) better theoretical understanding of the relation between appearance and development of flow pattern during the fireball evolution and the processes such as thermalization, creation of quark-gluon plasma, phase transitions, etc. [4–7], iii) the study of mean field effects [6,8], iv) the importance of flow for other measurements such as two-particle interferometry [9–13], v) development of new techniques suitable for flow study at high energies [14,15]. Although all forms of flow are interrelated and represent only different parts of one global picture, usually people discuss different forms of collective flow, such as longitudinal expansion, radial transverse expansion, directed flow, elliptic flow [15,8].

Flow introduces strong space-momentum correlation in the particle production. Particles with a given rapidity and transverse momentum are produced only by some part of the entire source. This part we call as an effective source. It moves with the rapidity close to that of the particles [9,11] in the midrapidity region, or slightly less than that for particles in the fragmentation region [13]. The subject of the current study is transverse (radial and directed) flow, namely the questions how azimuthally symmetric radial flow interferes with directed flow and affects the directed flow signal.

---

*On leave from Moscow Engineering Physics Institute, Moscow, 115409, Russia

It is convenient to characterize directed flow by $v_1$, the amplitude of the first harmonic in the Fourier decomposition of the particle azimuthal distribution [14]. Depending on which particular distribution is studied, the coefficients in the Fourier decomposition depend on different variables. If it is a 2-dimensional rapidity and azimuthal angle distribution, then $v_1$ depends only on rapidity, and $v_1 = \langle p_x \rangle / \langle p_t \rangle$, where $\langle p_t \rangle$ is the mean transverse momentum, and $\langle p_x \rangle$ is an another widely used quantity to describe flow, the mean projection of the transverse momentum onto the reaction plane. If one studies the production of particles with a given rapidity and transverse momentum, then

$$E\frac{d^3N}{d^3p} = \frac{d^3N}{p_t dp_t dy d\phi'} = \frac{1}{2\pi}\frac{d^2N}{p_t dp_t dy}(1 + 2v_1\cos(\phi') + 2v_2\cos(2\phi') + ...), \tag{1}$$

and $v_1$ depends on two variables $y$ and $p_t$. The dependence $v_1(p_t)$ for particles with a given rapidity is the main topic of the current study. We will show that the radial expansion of the source results in specific shapes of this function, which can be studied experimentally, providing an information on the directed as well as radial components of the effective source velocity.

First studies of event shape anisotropy as a function of transverse momentum [16,17] have shown very interesting results. Such a study using Fourier decomposition of the azimuthal distribution promises better quantitative description of the effect. One of the advantages would be the possibility to correct the results for the reaction plane resolution [2,14]. The analysis of $v_1(p_t)$ is to a large extend independent from the uncertainties in the $p_t$ dependence of spectrometer acceptance and efficiency, and in this sense such an analysis has preference in comparison with the analysis of triple differential distributions.

Transverse directed flow is a result of a movement of an effective source in the transverse plane (below we assume that this movement is along the "x" axis). In the source rest frame the first moment of azimuthal distribution ($v_1$) is zero. The final anisotropy (here and below we discuss only the first harmonic of the azimuthal distribution) appears only as a consequence of the source movement in the transverse direction. Below we derive a general expression for $v_1$, assuming that the invariant distribution in the source rest frame is known and (for simplicity) azimuthally symmetric:

$$E\frac{d^3N}{d^3p} \equiv J(\mathbf{p_t}, y) = J(p_t, y). \tag{2}$$

Source movement in the transverse direction results in $J \rightarrow J' = J(\mathbf{p_t}', y)$, where $\mathbf{p_t}'$ is defined by the Lorentz shift with velocity $\beta_a$ along the "x" axis. Assuming that the shift is small (the directed flow velocity expected to be $\beta_a \leq 0.1$), $p'_x = p_x - \beta_a E$, and taking into account that $\partial p_t / \partial p_x = \cos(\phi)$, one gets:

$$v_1 = \frac{\int d\phi J(\mathbf{p_t}', y)\cos(\phi)}{\int d\phi J(\mathbf{p_t}', y)} = -\frac{\beta_a E}{2}\frac{d \ln J(p_t, y)}{dp_t}. \tag{3}$$

A comparison of exact numerical calculations for the models discussed below with the results of application of this formula shows that for the values of $\beta_a \leq 0.1$, the formula (3) is accurate at the level of a few percent.

Below we apply the derived formula to a few particular cases, when the thermal source undergoes an isotropic expansion. We first consider an expansion in a non-relativistic case, then go to a relativistic generalization, and, finally, discuss a model of isotropically expanding thermal shell [18], widely used in the analysis of BEVALAC and SIS data [19,20]. Using simple models permits us to perform all calculations analytically to keep clear the qualitative features of the effect. Note that directed as well as isotropic expansion velocities are defined in the frame moving longitudinally with the rapidity of the effective source.

Directed flow of protons is the most pronounced and less affected by another effects such as shadowing. Protons are relatively heavy particles ( $m \gg T$, $m$ is the proton mass, $T$ is the temperature), and often can be treated as non-relativistic particles. This is certainly justified in the source rest frame, where the proton kinetic energy is of the order of temperature $E^* - m \sim T \ll m$. To have possibility to treat protons non-relativistically in the transverse direction in the *analysis* frame we have to assume that $m_t - m \ll m$, what restricts the non-relativistic consideration to the region of $p_t \leq 0.5\ GeV$.

The transverse isotropic expansion of the source can be described as a superposition of different sources moving radially with an expansion velocity $\beta_0$. Then the (non-relativistic) transverse momentum distribution of protons from a radially expanding thermal source can be written as:

$$\frac{1}{N}\frac{d^2N}{d\mathbf{p_t}} = \frac{1}{(2\pi)^2\,(2mT)}\int d\psi \exp\left(-\frac{(p_x - p_0\cos(\psi))^2 + (p_y - p_0\sin(\psi))^2}{2mT}\right), \tag{4}$$

where $p_0 = m\beta_0$. The integration over $\psi$ (the orientation of the expansion velocity) results in the distribution:

$$\frac{1}{N}\frac{d^2N}{d\mathbf{p_t}} = \frac{1}{2\pi(2mT)} \exp\left(-\frac{p_t^2+p_0^2}{2mT}\right) I_0(\xi), \tag{5}$$

where $I_0$ is the modified Bessel function, and $\xi = \beta_0 p_t/T$.

Using the formula (3) one gets an expression for $v_1$:

$$v_1(p_t) = \frac{p_t\beta_a}{2T}\left(1 - \frac{m\beta_0}{p_t}\frac{I_1(\xi)}{I_0(\xi)}\right). \tag{6}$$

In our analysis we work in the frame moving longitudinally with the effective source rapidity. In this frame the longitudinal particle momenta usually can be neglected, what was done in the derivation above. If one considers the particle production with rapidities far from the rapidity of the effective source, when particle longitudinal momenta are large, one should make a substitution $m \to \sqrt{m^2+p_l^2}$.

It follows from (6) that in the case without radial expansion $v_1$ linearly depends on the transverse momentum with the slope of $0.5\beta_a/T$. The radial expansion of the system decreases the directed flow signal changing also the shape of $v_1(p_t)$ in the low $p_t$ region. Remarkably, that at some parameter values, namely such that $m\beta_0^2/2 > T$ (for $T = 100\ MeV$ it corresponds to $\beta_0 > 0.46$), $v_1(p_t)$ has a region (at small $p_t$), where it is negative (see Fig. 1). Physically it corresponds to the case, when particle production with such a value of $p_t$ is more probable from the part of the effective source which moves to the opposite direction than the flow direction (in this case directed flow and "expansion" flow compensate each other).

The relativistic generalization of the above formulae is strait-forward, although results into somewhat less transparent expressions. The invariant distribution in this case can be written in the form:

$$\begin{aligned} J &\propto \int d\psi E^* e^{-E^*/T} \\ &= \int d\psi (E\cosh(y_t) - p_t\cos(\psi)\sinh(y_t)) \exp(-(E\cosh(y_t) - p_t\cos(\psi)\sinh(y_t))/T) \\ &= T e^{-\chi}(\chi I_0(\xi) - \xi I_1(\xi)), \end{aligned} \tag{7}$$

where $\chi = E\cosh(y_t)/T$, $\xi = p_t\sinh(y_t)/T$; $y_t = 0.5\ln((1+\beta_0)/(1-\beta_0))$ is the transverse rapidity; $I_0$ and $I_1$ are modified Bessel functions. Such a distribution yields:

$$v_1 = \frac{\beta_a p_t\cosh(y_t)}{2T}\left[1 - \frac{I_1(\xi)E^2\sinh(y_t)\cosh(y_t) + I_0(\xi)(p_t T\cosh(y_t) - ET\sinh(y_t)^2)}{p_t T\cosh(y_t)(\chi I_0(\xi) - \xi I_1(\xi))}\right]. \tag{8}$$

Note, that the energy $E$ is measured in the frame moving longitudinally with the same velocity as an effective source. Studying particle production close to the fragmentation region, one should remember, that the rapidity of the effective source could be less than that of the particles [13]. In this case $E = m_t \cosh(y^* - y)$, where $y^*$ is the effective source rapidity.

If the particles are emitted from a thermalized *spherical shell* at temperature $T$, expanding with velocity $\beta_r$ (we use different notation to distinguish $\beta_r$ from $\beta_0$, the expansion velocity in the transverse *plane*), the expected invariant distribution has the form [18,19]:

$$J \propto \exp(-E\gamma/T) \left[ \frac{\sinh(\alpha)}{\alpha}(E + T/\gamma) - \cosh(\alpha)T/\gamma \right], \tag{9}$$

where $\gamma = 1/\sqrt{1-\beta_r^2}$, and $\alpha = \beta_r p \gamma / T$. An application of the prescription (3) gives:

$$\begin{aligned} v_1 = \frac{p_t \beta_a \gamma}{2T} \Big\{ & E\sinh(\alpha)/\alpha - \cosh(\alpha)T/\gamma + [(E + T/\gamma)(\cosh(\alpha)/\alpha - \sinh(\alpha)/\alpha^2) \\ & - \sinh(\alpha)T/\gamma]\beta_r E/p \} \, / \, \{(E + T/\gamma)\sinh(\alpha)/\alpha - \cosh(\alpha)T/\gamma\} \end{aligned} \tag{10}$$

Qualitatively this model predicts the same dependencies $v_1(p_t)$ as the model with the expansion only in the transverse plane. But the value of the expansion velocity needed in this model for $v_1(p_t)$ exhibits a dip at low $p_t$ is about 15% larger due to the 3-dimensional type of the expansion.

Summarizing, it is shown that a study of the dependence of the amplitude of the first harmonic in the azimuthal distributions on transverse momentum reveals important features of directed and transverse radial flow. The magnitude and the shape of $v_1(p_t)$ is very sensitive to such parameters of the system as an effective temperature, radial expansion and directed flow velocities. Strong radial expansion (approximately $\beta_0 \geq \sqrt{2T/m}$) would evidence itself in the directed flow signal showing an anti-flow of the particles with low transverse momentum. The preliminary results of E877 Collaboration presented at HIP-AGS '96 Workshop [21] suggests that such a regime could be the case in gold-gold collisions at the AGS.

The author acknowledge the discussions with W.-C. Chang.

[1] E877 Collaboration, J. Barrette *et al.*, Phys. Rev. Lett. **70**, 2996(1993).

[2] E877 Collaboration, J. Barrette *et al.*, preprint nucl-ex/9610006, submitted to Phys. Rev. C; T. Hemmick for the E877 Collaboration, talk at QM '96, Nucl. Phys. **A610** (1996), in print.

[3] T. Wienold for the NA49 Collaboration, talk at QM '96. Nucl. Phys. **A610** (1996), in print.

[4] J. Stachel, talk at QM '96, Nucl. Phys. **A610** (1996), in print.

[5] L. Bravina, L.P. Csernai, P. Lévai, and D. Strottman, Phys. Rev. C **50**, 2161 (1994); L. Bravina, Phys. Lett. B **344**, 49 (1995).

[6] B.A. Li, C.M. Ko, Phys. Rev. C **52**, 2037 (1995); **53**, R22 (1996).

[7] C.M. Hung and E.V. Shuryak, Phys. Rev. Lett. **75**,4003 (1995).

[8] H. Sorge, Preprint SUNY-NTG 96-40, nucl-th/9610026, 1996.

[9] U. Heinz, preprint nucl-th/960929, to be published in the NATO ASI Proceedings Series by Plenum Publ. Corp. (M.N. Harakeh, O. Scholten, and J.K. Koch, eds.) .

[10] U. A. Wiedemann and U. Heinz, preprint nucl-th/9610043.

[11] T. Csörgő, B. Lorstad, Phys. Rev. C **54**, 1390 (1996).

[12] S.A. Voloshin and W.E. Cleland, Phys. Rev. C **53**, 896 (1996); Phys. Rev. C **54**, ??? (1996).

[13] D. Miśkowiec for E877 Collaboration, talk at QM '96, Nucl. Phys. **A610** (1996), in print.

[14] S. Voloshin and Y. Zhang, Z. Phys. C **70**, 665 (1996);

[15] J. Y. Ollitrault, Phys. Rev. **D46**, 229 (1992); Phys. Rev. **D48** 1132 (1993).

[16] Y. Zhang and J.P. Wessels for E877 Collaboration, Nucl. Phys. **A590**, 557c (1995).

[17] B.A. Li, C.M. Ko, G.Q. Li, Phys. Rev. C **54**, 844 (1996).

[18] P.J. Siemens and J.O. Rasmussen Phys. Rev. Lett. **42**, 880(1978).

[19] EOS Collaboration, S. Wang *et al.*, Phys. Rev. Lett. **76**, 3911 (1996); M.A. Lisa *et al.*, Phys. Rev. Lett. **75**, 2662 (1995);

[20] N. Herrmann for the FOPI Collaboration, talk at QM '96, Nucl. Phys. **A610** (1996), in print.

[21] W.C. Chang for the E877 Collaboration, Proc. Workshop "Heavy Ion Physics at the AGS '96", Wayne State University, 1996, in print.

## I. FIGURE CAPTION

FIG. 1. $v_1(p_t)$: solid lines – Eq. (6), dashed lines – Eq. (8). $T = 0.1\ GeV$, $\beta_a = 0.1$.

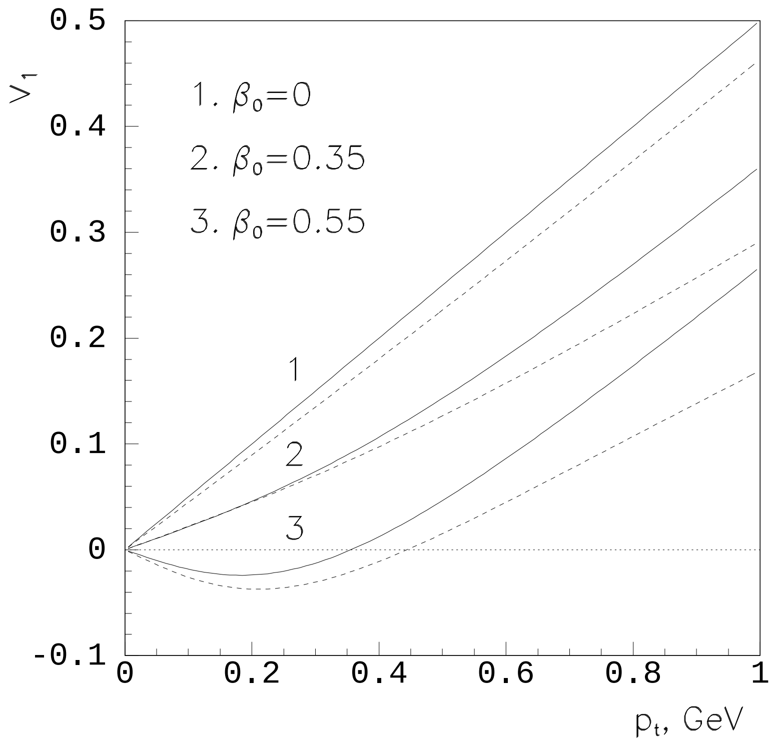


FIG. 1. $v_1(p_t)$: solid lines – Eq. (6), dashed lines – Eq. (8). $T = 0.1\ GeV$, $\beta_a = 0.1$.